Towards a Co-Design Framework for Future Mobility Systems

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Motivation

Motivation

$55 billion market in 2019, $557 billion in 2026

48 AV companies in California, 80+ in US

− 44% parking space,
− 66% emissions,
− 30% travel time
− 90% fatalities

Lack of specifications on their intended service

How performant should the AVs be?

What is the best fleet size?

How will AVs affect future public transportation systems?

Will the outcome be socially, economically, and environmentally sustainable?

The design of AVs and the design of AVs-enabled mobility systems are closely coupled.

Scope

We develop a co-design framework to solve the problem of designing and deploying an inter-modal Autonomous Mobility-on-Demand system, optimizing for

- its performance,
- the costs it produces, and
- its environmental footprint.
### Problem Setting – What Do We Want To Co-Design?

#### Autonomous Vehicles
- The vehicle autonomy.
- The AVs fleet size.

#### Public Transportation
- The public transit service frequency.

#### Public Roads
- The parking space allocation.
AMoD Systems
Autonomous Mobility-on-Demand systems for future urban mobility [Pavone, 2015]

AMoD Systems Design
Solving a multi-periodic stochastic model of the rail-car fleet sizing by two-stage optimization formulation [Sayarshad et al., 2010]

Urban Parking for AVs
Parking spaces in the age of shared autonomous vehicles [Zhang et al., 2017]

- No joint design of AVs and AVs-enabled mobility systems.
- No flexible toolboxes.
- Not directly useful for stakeholders.

- No design considerations
  Lay the foundations for our framework

- No AVs-specific characteristics
  Problem-specific structure, non-modular, single solution

- Not connected with the AVs-enabled mobility system design
  Not considering multiple functions of parking space
We need a framework that allows to structure the design problem in a **modular** and **compositional** way.

**Co-Design**

A mathematical theory of Co-Design [Censi, 2015]

Monotone Co-Design problems; or, everything is the same [Censi, 2016]

A class of Co-Design problems with cyclic constraints and their solution [Censi, 2017]

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**Offers a mathematical formalization of Co-Design problems**

**Provides modularity and compositionality**
Co-Design

Abstraction of a Design Problem

A Design Problem (DP) is abstracted as a monotone map $h$ between provided functionalities and the antichain of requires resources (posets $\langle F, \leq_F \rangle$ and $\langle R, \leq_R \rangle$).

Abstraction of a Co-Design Problem

A Co-Design Problem (CDP) is abstracted as an interconnection of individual DPs.

Co-Design Goal

Find the antichain of all rational resources $r_1, \ldots, r_N \in R$ which provide a given functionality $f \in F$. 
Modeling – Network Flow Model for Intermodal AMoD

Assumptions

- Network flow approach
- Time-invariant model.

Mesoscopic
System-level planning perspective

\[ G_R = (V_R, A_R) \]
\[ G_W = (V_W, A_W) \]
\[ G_P = (V_P, A_P) \]

Graph

Mode-switching arcs: \( A_C \subseteq V_R \times V_W \cup V_W \times V_R \cup V_P \times V_W \cup V_W \times V_P \).

Extended graph: \( A = A_W \cup A_R \cup A_P \cup A_C, V = V_R \times V_W \times V_P \).
Modeling – Network Flow Model for Intermodal AMoD

Travel Requests

Assume $M$ travel requests $\rho_m = (o_m, d_m, \alpha_m) \in V_w \times V_w \times \mathbb{R}_+$, $\forall m \in \{1, \ldots, M\}$.

Constraints

$$\sum_{i:(i,j) \in A} f_m(i,j) + \mathbb{I}_{j=\o_m} \cdot \alpha_m = \sum_{k:(j,k) \in A} f_m(j,k) + \mathbb{I}_{j=\d_m} \cdot \alpha_m, \quad \forall m \in M, j \in V$$

$$\sum_{i:(i,j) \in A_R} \left( f_0(i,j) + \sum_{m \in M} f_m(i,j) \right) = \sum_{k:(j,k) \in A_R} \left( f_0(j,k) + \sum_{m \in M} f_m(j,k) \right), \quad \forall j \in V_R$$

$f_m(i,j) \geq 0, \quad \forall m \in M, (i,j) \in A$

$f_0(i,j) \geq 0, \quad \forall (i,j) \in A_R$. 
Modeling – Travel Time and Speed

Road

- Each \((i, j) \in A_R\) has a speed limit \(v_{L,ij}\).
- AVs safety protocols impose a maximum achievable speed \(v_a\).
- Too slow AVs are dangerous: \((i, j)\) is kept in \(A_R\) iff \(v_a \geq \beta \cdot v_{L,ij}, \quad \beta \in (0, 1]\).
- Then, \(v_{ij} = \min\{v_a, v_{L,ij}\}\).

Pedestrians

Constant walking speed \(v_{ij}\) for each \((i, j) \in A_W\).

Public Transportation System

- The public transit system at node \(j\) operates with frequency \(\varphi_j\).
- Switching from \(j\) to a pedestrian vertex \(i\) takes \(t_{WP}\): \(t_{ij} = t_{WP} + \frac{1}{\varphi_j} \forall (i, j) \in A_P\).
Modeling – Properties

Energy Consumption

**AVs:**
- Urban driving cycle.
- \( e_{ij} = e_{\text{cycle}} \cdot \frac{s_{ij}}{s_{\text{cycle}}} \ \forall (i, j) \in A_R. \)

**Public Transportation:**
- Assumption: Customers-independent operation.
- Constant energy consumption per unit time.

AVs Fleet Size

\[
n_{v,e} = \sum_{(i,j) \in A_R} \left( f_0(i,j) + \sum_{m \in M} f_m(i,j) \right) \cdot t_{ij} \leq n_{v,\text{max}}.\]
Co-Design – The Monotone Co-Design Problem

**Figure 2**: Schematic representation of the individual design problems (a-c) as well as of the co-design problem of the full system (d). In solid green the provided functionalities and in dashed red the required resources. The edges in the co-design diagram (d) represent co-design constraints: The resources required by a first design problem are the lower bound for the functionalities provided by the second one.

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Co-Design – AVs

AVs Design Problem

We design the maximal achievable speed $v_a$.

**Functionality:**
- Maximal achievable speed $v_a$.
- $\mathcal{F}_v = \mathbb{R}_+$ (in mph).

**Resources:**
- Vehicle fixed costs $C_{v,f} = C_{v,v} + C_{v,a}$.
- Vehicle operational costs $C_{v,o}$.
- $\mathcal{R}_v = \mathbb{R}_+ \times \mathbb{R}_+$ (in USD $\times$ USD/mile).

**Functionality/Resources Relation**
- Higher speed, more advanced technology.
- $v_a$ as monotone function of costs.
**Co-Design – Public Transportation System**

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**Subway Design Problem**

**We design the service frequency** $\varphi_j$, assuming

$$
\frac{\varphi_j}{\varphi_{j,\text{baseline}}} = \frac{n_s}{n_{s,\text{baseline}}}
$$

**Functionality:**

- Acquired trains $n_{s,a} = n_s - n_{s,\text{baseline}}$.
- $F_s = N$.

**Resources:**

- Train fleet fixed costs $C_{s,f}$.
- Train fleet operational costs $C_{s,o}$.
- $R_s = \mathbb{R}_+ \times \mathbb{R}_+$ (in USD $\times$ USD/year).

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**Functionality/Resources Relation**

- More trains, higher fixed costs.
- More trains require more operators: higher operational costs.
## I-AMoD Optimization Framework Design Problem

### Functionality:
- Demand satisfaction: 
  \[ \alpha_{\text{tot}} := \sum_{m \in \mathcal{M}} \alpha_m. \]
- \( \mathcal{F}_0 = \mathbb{R}_+ \).

### Resources:
- Maximal achievable speed \( v_a \).
- Available AVs per fleet \( n_{v,\text{max}} \).
- Acquired trains \( n_{s,a} \).
- Average travel time per trip: 
  \[ t_{\text{avg}} := \frac{1}{\alpha_{\text{tot}}} \sum_{m \in \mathcal{M}, (i,j) \in \mathcal{A}} t_{ij} \cdot f_m(i,j), \]
- Total AVs-driven distance: 
  \[ s_{v,\text{tot}} := \sum_{(i,j) \in \mathcal{A}_R} s_{ij} \cdot \left( f_0(i,j) + \sum_{m \in \mathcal{M}} f_m(i,j) \right). \]
- AVs emissions: 
  \[ m_{\text{CO}_2,v,\text{tot}} := \gamma \sum_{(i,j) \in \mathcal{A}_R} e_{ij} \cdot \left( f_0(i,j) + \sum_{m \in \mathcal{M}} f_m(i,j) \right). \]
- \( \mathcal{R}_0 = \mathbb{R}_+ \times \mathbb{N} \times \mathbb{N} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \).
Co-Design – I-AMoD Optimization Framework

\[ \alpha_{\text{tot}} \]

\begin{align*}
\text{I-AMoD} & \quad \text{s}_{v,\text{tot}} \quad n_{s,a} \quad v_a \quad n_{v,\text{max}} \quad t_{\text{avg}} \quad mCO_2,v,\text{tot} \\
\end{align*}

Functionality/Resources Relation

\[
\min_{\{f_m(\cdot,\cdot)\}_m, f_0(\cdot,\cdot)} t_{\text{avg}} = \frac{1}{\alpha_{\text{tot}}} \sum_{m \in M, (i,j) \in A} t_{ij} \cdot f_m(i,j), \quad \text{s.t.} \; (1), (2), (3).
\]

- (2): Road congestion.
- (3): Fleet limitations.
Co-Design – The Monotone Co-Design Problem

Full Co-Design Problem

**Functionality:**
- Demand satisfaction $\alpha_{\text{tot}}$
- $\mathcal{F} = \mathbb{R}_+$

**Resources:**
- Total costs $C_{\text{tot}} = C_v + C_s$, with
  - $C_v = \frac{C_{v,f}}{l_v} \cdot n_v + C_{v,o} \cdot s_{v,\text{tot}}$
  - $C_s = \frac{C_{s,f}}{l_s} \cdot n_{s,a} + C_{s,o}$
- Average travel time per trip $t_{\text{avg}}$
- Total emissions: $m_{\text{CO}_2,\text{tot}} = m_{\text{CO}_2,v,\text{tot}} + m_{\text{CO}_2,s} \cdot n_s$
- $\mathcal{R} = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$
Co-Design – The Monotone Co-Design Problem

FIGURE 2: Schematic representation of the individual design problems (a-c) as well as of the co-design problem of the full system (d). In solid green the provided functionalities and in dashed red the required resources. The edges in the co-design diagram (d) represent co-design constraints: The resources required by a first design problem are the lower bound for the functionalities provided by the second one.

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Case Study – Washington DC, USA

**Dataset Construction**

Road network: OpenStreetMap.  
Public Transit network: GTFS.  
Origin-destination pairs: WMATA.  
Demand: 15,872 travel requests → 24.22 requests/s

**Co-Design**

Subway frequency: \{100\%, 133\%, 200\%\}.  
AVs speed: \(v_a \in \{20 \text{ mph}, 25 \text{ mph}, \ldots, 50 \text{ mph}\}\).  
Fleet size: \(n_{v,\text{max}} \in \{0, 500, \ldots, 6000\}\).
## Problem Setting

Case Study – Parameters and Units for Sensitivity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
<th>Case 1</th>
<th>Case 2.1</th>
<th>Case 2.2</th>
<th>Case 3.1</th>
<th>Case 3.2</th>
<th>Units</th>
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<tbody>
<tr>
<td>Baseline road usage</td>
<td>$u_{ij}$</td>
<td>93</td>
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<td>%</td>
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<tr>
<td>Vehicle operational cost</td>
<td>$C_{v,o}$</td>
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<td>0.084</td>
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<td>USD/km</td>
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<tr>
<td>Vehicle cost</td>
<td>$C_{v,v}$</td>
<td>32,000</td>
<td>32,000</td>
<td>26,000</td>
<td>32,000</td>
<td>32,000</td>
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<td>USD/car</td>
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<tr>
<td>20 mph</td>
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<td>15,000</td>
<td>20,000</td>
<td>3,700</td>
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<td>500,000</td>
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<td>USD/car</td>
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<td>25 mph</td>
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<td>55,000</td>
<td>6,200</td>
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<td>500,000</td>
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<td>USD/car</td>
</tr>
<tr>
<td>35 mph</td>
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<td>8,700</td>
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<td>40 mph</td>
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<td>15,000</td>
<td>115,000</td>
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<td>500,000</td>
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<td>USD/car</td>
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<td>45 mph</td>
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<td>15,000</td>
<td>130,000</td>
<td>12,000</td>
<td>0</td>
<td>500,000</td>
<td></td>
<td>USD/car</td>
</tr>
<tr>
<td>50 mph</td>
<td></td>
<td>15,000</td>
<td>150,000</td>
<td>13,000</td>
<td>0</td>
<td>500,000</td>
<td></td>
<td>USD/car</td>
</tr>
<tr>
<td>Vehicle life</td>
<td>$l_t$</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
<td>years</td>
</tr>
<tr>
<td>CO$_2$ per Joule</td>
<td>$\gamma$</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td></td>
<td>$g/J$</td>
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<td>Time from $W_R$ to $R$</td>
<td>$t_{WR}$</td>
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<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td>s</td>
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<tr>
<td>Time from $R$ to $W_R$</td>
<td>$t_{RW}$</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td></td>
<td>s</td>
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<tr>
<td>Speed limit fraction</td>
<td>$\beta$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Subway operational cost</td>
<td>$C_{s,o}$</td>
<td>100%</td>
<td>148,000,000</td>
<td></td>
<td></td>
<td></td>
<td>USD/year</td>
<td></td>
</tr>
<tr>
<td>Subway fixed cost</td>
<td>$C_{s,f}$</td>
<td>133%</td>
<td>197,000,000</td>
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<td></td>
<td></td>
<td>USD/year</td>
<td></td>
</tr>
<tr>
<td>200%</td>
<td></td>
<td>295,000,000</td>
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<td></td>
<td></td>
<td>USD/year</td>
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<td></td>
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<tr>
<td>Train life</td>
<td>$l_t$</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>years</td>
</tr>
<tr>
<td>Subway CO$_2$ emissions per train</td>
<td>$m_{CO_2,s}$</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ton/yr</td>
</tr>
<tr>
<td>Train fleet baseline</td>
<td>$n_s,baseline$</td>
<td>112</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>trains</td>
</tr>
<tr>
<td>Subway service frequency</td>
<td>$\varphi_s,baseline$</td>
<td>$1/6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>s/minute</td>
</tr>
<tr>
<td>Time from $W_R$ to $F_P$ and vice-versa</td>
<td>$t_{WS}$</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>s</td>
</tr>
</tbody>
</table>
We consider an emissions penalty of 40 USD/kg [Howard et al., 2015]
Results – Case 1 (Constant Automation Cost)
Autonomous cars could cut traffic and pollution — or make them worse, planners say
(Washington Post, 2019)

Automated vehicles can’t save cities
(New York Times, 2018)

Autonomous vehicles: To park or not to park?
(Forbes, 2019)
Extension – Parking Space Allocation

Consider travel requests in two **consecutive** time windows:

\[ \rho_{m_1} = (o_{m_1}, d_{m_1}, \alpha_{m_1}) \, \forall m_1 \in \{1, \ldots, M_1\}, \quad \eta_{m_2} = (o_{m_2}, d_{m_2}, \alpha_{m_2}) \, \forall m_2 \in \{1, \ldots, M_2\}. \]

Look at the flow changes on road arcs:

\[ \Delta \text{Flow}(i, j) = |f_0(i, j) - g_0(i, j) + f_{m_1}(i, j) - g_{m_2}(i, j)| \quad \forall (i, j) \in A_R. \]

Distribute parked cars accordingly:

\[ n_p(i, j) = \frac{\Delta \text{Flow}(i, j)}{\sum_{(i, j) \in A_R} \Delta \text{Flow}(i, j)} \cdot (n_{v, \text{max}} - n_{v, \text{e}}). \]
Extension – Parking Space Allocation

Figure 1: Co-design problem of the full system.

Figure 2: Schematic representation of the individual design problems (a-c) as well as of the co-design problem of the full system (d). In solid green the provided functionalities and in dashed red the required resources. The edges in the co-design diagram (d) represent co-design constraints: The resources required by a first design problem are the lower bound for the functionalities provided by the second one.
Extension – Case 1 Preliminary Results

![Graph showing the relationship between total cost and average travel time.](Graphic)

**Max**
- 50 mph
- 30 mph
- 20 mph
- 0 mph

**Min**
- 50 mph
- 30 mph
- 20 mph
- 0 mph

- 12 trains
- 37 trains
- 2000 AVs
- 5000 AVs

![Bar chart comparing costs and travel times for different scenarios.](Graphic)
Extension – Case 1 Preliminary Results (musical chairs)
Conclusions

Summary

- Co-Design framework for future mobility systems.
- Provides a new, different perspective.
- Tool for stakeholders such as AVs companies and policy makers.
- Modular and compositional, ready to be extended.

Outlook

- Parking space allocation.
- User-friendly interface.
- Model complexity.
Partial Orders

Consider a set $\mathcal{P}$ and a partial order $\preceq_{\mathcal{P}}$, defined as a reflexive, antisymmetric, and transitive relation. Then, $\mathcal{P}$ and $\preceq_{\mathcal{P}}$ define the partially ordered set (poset) $\langle \mathcal{P}, \preceq_{\mathcal{P}} \rangle$.

Bottom and Top

The least and maximum elements of a poset are called bottom and top, and are denoted by $\bot_{\mathcal{P}}$ and $\top_{\mathcal{P}}$, respectively.

CPO and DCPO

A set $S \subseteq \mathcal{P}$ is directed if each pair of elements $x, y \in S$ has an upper bound. A poset is a directed complete partial order (DCPO) if each of its directed subsets has a top, and it is a complete partial order (CPO) if it has a bottom as well.
Chains and Antichains

A *chain* is a subset $S \subseteq \mathcal{P}$ where all elements are comparable, i.e., for $x, y \in S$, $x \preceq \mathcal{P} y$ or $y \preceq \mathcal{P} x$. Conversely, an *antichain* is a subset $S \subseteq \mathcal{P}$ where no elements are comparable, i.e., for $x, y \in S$, $x \preceq \mathcal{P} y$ implies $x = y$.

Monotonicity

A map $g : \mathcal{P} \rightarrow \mathcal{Q}$ between two posets is *monotone* iff $x \preceq \mathcal{P} y$ implies $g(x) \preceq \mathcal{Q} g(y)$.

Scott Continuity

A map $f : \mathcal{P} \rightarrow \mathcal{Q}$ between directed complete partial orders (DCPOs) is *Scott Continuous* iff for each directed subset $D \subseteq \mathcal{P}$, the image $f(D)$ is directed, and $f(\text{sup}(D)) = \text{sup} f(D)$. 
Background – Co-Design

Least Fixed Point

A **least fixed point** of $f : \mathcal{P} \to \mathcal{P}$ is the minimum (if it exists) of the set of fixed points of $f$:

$$\text{lfp}(f) = \min\{x \in \mathcal{P} : f(x) = x\}.$$  

The least fixed point does not need to exist. Monotonicity of the map $f$ plus completeness is sufficient to ensure existence.

Lemma

- If $\mathcal{P}$ is a CPO and $f : \mathcal{P} \to \mathcal{P}$ is monotone, then lfp$(f)$ exists.
- Assume $\mathcal{P}$ is a CPO, and $f : \mathcal{P} \to \mathcal{P}$ is Scott continuous, then the least fixed point of $f$ is the supremum of the Kleene ascent chain

$$\bot \leq f(\bot) \leq f(f(\bot)) \leq \ldots \leq f^n(\bot) \leq \ldots.$$
Design Problem

A design problem (DP) is a tuple \( \langle F, R, h \rangle \) such that \( F \) and \( R \) are CPOs, and \( h : F \to AR \) is a **monotone** and **Scott-continuous** function. Each functionality \( f \in F \) corresponds to an antichain of resources \( h(f) \in AR \).

Monotone Co-Design Problem

A MCDP is a tuple \( \langle A, T, v \rangle \), where:

- \( A \) is any set of atoms, to be used as labels.
- The term \( T \) in the \{series, par, loop\} algebra describes the structure of the graph:
  \[
  T \in \text{Terms}\left(\{\text{series, par, loop}\}, A\right).
  \]
- The valuation: \( v : A \to \text{DP} \) assigns a DP to each atom.
Product Operator
For two maps $h_1 : \mathcal{F}_1 \rightarrow A\mathcal{R}_1$ and $h_2 : \mathcal{F}_2 \rightarrow A\mathcal{R}_2$, define

$$h_1 \otimes h_2 : (\mathcal{F}_1 \times \mathcal{F}_2) \rightarrow A(\mathcal{R}_1 \times \mathcal{R}_2)$$

$$\langle f_1, f_2 \rangle \mapsto h_1(f_1) \times h_2(f_2).$$

Series Operator
For two maps $h_1 : \mathcal{F}_1 \rightarrow A\mathcal{R}_1$ and $h_2 : \mathcal{F}_2 \rightarrow A\mathcal{R}_2$, if $\mathcal{R}_1 = \mathcal{F}_2$, define

$$h_1 \circ h_2 : \mathcal{F}_1 \rightarrow A\mathcal{R}_2$$

$$h_1 \mapsto \text{Min} \bigcup_{r_1 \in h_1(f)} h_2(r_1).$$
Loop Operator

For a map $h : \mathcal{F}_1 \times \mathcal{F}_2 \to \mathcal{A} \mathcal{R}$, define

$$h^\dagger : \mathcal{F}_1 \to \mathcal{A} \mathcal{R},$$

$$f_1 \mapsto \text{lfp} \left( \psi_{f_1}^h \right),$$

where lfp is the least-fixed point operator, and $\psi_{f_1}^h$ is

$$\psi_{f_1}^h : \mathcal{A} \mathcal{R} \to \mathcal{A} \mathcal{R},$$

$$R \mapsto \text{Min} \bigcup_{r \in R} h(f_1, r) \cap \uparrow r.$$
A design problem with implementation (DPI) is a tuple \( \langle \mathcal{F}, \mathcal{R}, \mathcal{I}, \text{exec}, \text{eval} \rangle \), where

- \( \mathcal{F} \) is a poset, called **functionality** space.
- \( \mathcal{R} \) is a poset, called **resources** space.
- \( \mathcal{I} \) is a poset, called **implementation** space.
- the map \( \text{exec} : \mathcal{I} \to \mathcal{F} \), execution, maps an implementation to the functionality it provides.
- the map \( \text{eval} : \mathcal{I} \to \mathcal{R} \), evaluation, maps an implementation to the resource it requires.
Background – Co-Design

**Problem**

Given a functionality $f \in \mathcal{F}$, find the implementations in $\mathcal{I}$ that realize the functionality $f$ (or higher) with minimal resources, or provide a proof that there are none:

$$
\begin{aligned}
\text{using} & \quad i \in \mathcal{I}, \\
\text{Min} & \quad r, \\
\text{s.t.} & \quad r = \text{eval}(i), \\
& \quad f \preceq \mathcal{F} \text{ exec}(i).
\end{aligned}
$$
Problem

Given a DPI $\langle F, R, I, \text{exec}, \text{eval} \rangle$, define the map $h : F \rightarrow AR$ that associates to each functionality $f$ the objective function of Problem 1, which is the set of minimal resources necessary to realize $f$:

$$h : F \rightarrow AR,$$

$$f \mapsto \text{Min}_{\leq R} \{ \text{eval}(i) | (i \in I) \land (f \leq \text{exec}(i)) \}.$$

If a certain functionality $f$ is infeasible, then $h(f)$ is the empty set.
Suppose $d_p_0 = \text{loop}(d_p_0)$, where $d_p_0$ is an MCDP that is described only using series and parallel operators. Suppose that the resource space is $\mathcal{R}_0$. Then evaluating $h_0$ takes at most $c$ computation:

- **Memory:** $O(\text{width}(\mathcal{R}_0))$.
- **Number of steps:** $O(\text{height}(A\mathcal{R}_0))$.
- **Computation:** $O(\text{width}(\mathcal{R}_0) \times \text{height}(A\mathcal{R}_0) \times c)$
Problem

Each road arc is subject to a baseline usage $u_{ij}$ and has a nominal capacity $c_{ij}$.

$$f_0(i, j) + \sum_{m \in M} f_m(i, j) + u_{ij} \leq c_{ij} \quad \forall (i, j) \in A_R.$$
Co-Design – The Monotone Co-Design Problem
# Case Study – Parameters and Units for Sensitivity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline road usage</td>
<td>( u_{ij} )</td>
<td>93</td>
<td>%</td>
</tr>
<tr>
<td>Vehicle operational cost</td>
<td>( C_{v,o} )</td>
<td>0.084</td>
<td>USD/mile</td>
</tr>
<tr>
<td>Vehicle cost</td>
<td>( C_{v,v} )</td>
<td>32,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>20 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>25 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>30 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>35 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>40 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>45 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>50 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>Vehicle automation cost</td>
<td>( C_{v,a} )</td>
<td>0.084</td>
<td>USD/car</td>
</tr>
<tr>
<td>35 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>40 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>45 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>50 mph</td>
<td></td>
<td>15,000</td>
<td>USD/car</td>
</tr>
<tr>
<td>Vehicle life</td>
<td>( l_i )</td>
<td>5</td>
<td>years</td>
</tr>
<tr>
<td>CO2 per Joule</td>
<td>( \gamma )</td>
<td>0.14</td>
<td>g/kJ</td>
</tr>
<tr>
<td>Time from ( G_W ) to ( G_R )</td>
<td>( t_{WR} )</td>
<td>300</td>
<td>s</td>
</tr>
<tr>
<td>Time from ( G_R ) to ( G_W )</td>
<td>( t_{RW} )</td>
<td>60</td>
<td>s</td>
</tr>
<tr>
<td>Speed limit fraction</td>
<td>( \beta )</td>
<td>( \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>Subway operational cost</td>
<td>( C_{s,o} )</td>
<td>148,000,000</td>
<td>USD/year</td>
</tr>
<tr>
<td>Subway fixed cost</td>
<td>( C_{s,f} )</td>
<td>14,500,000</td>
<td>USD/train</td>
</tr>
<tr>
<td>Train life</td>
<td>( l_t )</td>
<td>30</td>
<td>years</td>
</tr>
<tr>
<td>Subway CO2 emissions per train</td>
<td>( m_{CO2,s} )</td>
<td>140</td>
<td>ton/yr</td>
</tr>
<tr>
<td>Train fleet baseline</td>
<td>( n_{s,baseline} )</td>
<td>112</td>
<td>trains</td>
</tr>
<tr>
<td>Subway service frequency</td>
<td>( \varphi_{s,baseline} )</td>
<td>( \frac{1}{5} )</td>
<td>min/minute</td>
</tr>
<tr>
<td>Time from ( G_W ) to ( G_F ) and vice-versa</td>
<td>( t_{WS} )</td>
<td>60</td>
<td>s</td>
</tr>
</tbody>
</table>

## Table 1: Parameters, variables, numbers, and units for the case studies.

To account for the large presence of ride-hailing companies, we scale the taxi demand rate by a factor of 3. Overall, the demand dataset includes 15,872 travel requests, corresponding to a demand rate of 24.22 vehicles per hour, 25% of which are for trips longer than 2 miles, 100% of which are for trips longer than 2 miles, and 33% of which are for trips longer than 2 miles.

As discussed in Section 4, we design the system by means of subway service frequency, \( A_{Vs} \), and the Washington Metropolitan Area Transit Authority (WMATA) and Zardini, Lanzetti, Salazar, Censi, Frazzoli, and Pavone.

Towards a Co-Design Framework for Future Mobility Systems

6th September, 2019 | Giiole Zardini | zardini@stanford.edu
Results – Case 2.1 (speed-dependent automation cost)

+5% AVs fleet size
−10% AVs speed
+7% train fleet
Results – Case 2.2 (speed-dependent automation cost)
Results – Case 3.1 (no automation cost)
Results – Case 3.2 (high automation cost)
Results – Sensitivity Analysis

![Graph showing sensitivity analysis results with different cases labeled: Case 1, Case 2.1, Case 2.2, Case 3.1, Case 3.2. The x-axis represents $C_{tot}$ [Mil USD/month] and the y-axis represents $t_{avg}$ [min].]