Towards a Co-Design Framework for Future Mobility Systems

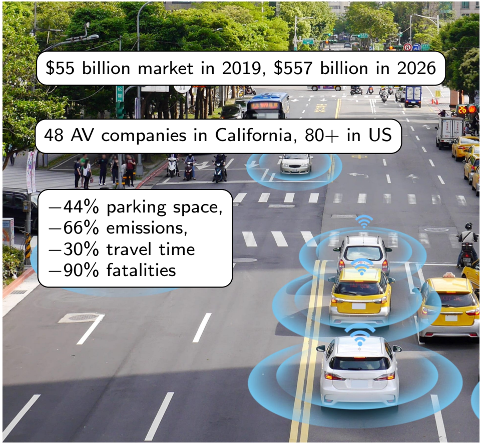
Gioele Zardini[1,2] Andrea Censi[2] Emilio Frazzoli [2] Marco Pavone[1]

[1]Autonomous Systems Lab (ASL)
Stanford University

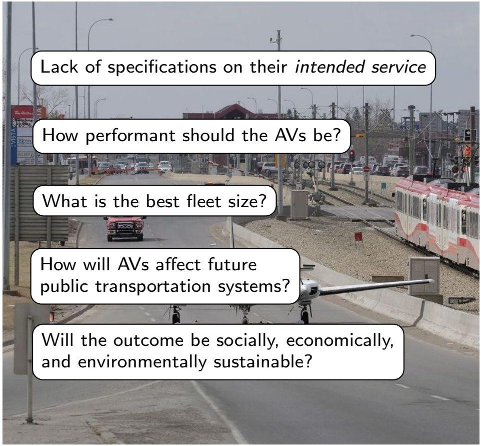
[2]Institute for Dynamic Systems and Control (IDSC)
ETH Zürich

6th September, 2019

Motivation



Data from: Allied Market Research, Aptiv 2018 Report, McKinsey & Company.



Motivation

The design of AVs and the design of AVs-enabled mobility systems are closely coupled.

Scope

We develop a **co-design** framework to solve the problem of *designing* and *deploying* an inter-modal Autonomous Mobility-on-Demand system, optimizing for

- its performance,
- the costs it produces, and
- its environmental footprint.

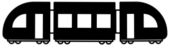
Problem Setting – What Do We Want To Co-Design?

Autonomous Vehicles

- The vehicle autonomy.
- The AVs fleet size.

Public Transportation

- The public transit service frequency.



Public Roads

- The parking space allocation.

Literature Review

AMoD Systems

Autonomous Mobility-on-Demand systems for future urban mobility [Pavone, 2015]

No design considerations

Lay the foundations for our framework

AMoD Systems Design

Solving a multi periodic stochastic model of the rail-car fleet sizing by two-stage optimization formulation

Joint design of multimodal transit networks and shared autonomous mobility fleets [Pinto et al., 2019]

No AVs-specific characteristics

Problem-specific structure, non-modular, single solution

Urban Parking for AVs

Parking spaces in the age of shared autonomous vehicles [Zhang et. al, 2017]

Not connected with the AVs-enabled mobility system design

Not considering multiple functions of parking space

- **No joint design of AVs and AVs-enabled mobility systems.**
- **No flexible toolboxes.**
- **Not directly useful for stakeholders.**

Literature Review

We need a framework that allows to structure the design problem in a **modular** and **compositional** way.

Co-Design

A mathematical theory of Co-Design [Censi, 2015]

Monotone Co-Design problems; or, everything is the same [Censi, 2016]

A class of Co-Design problems with cyclic constraints and their solution [Censi, 2017]

Offers a mathematical formalization of Co-Design problems

Provides modularity and compositionality

Co-Design

Abstraction of a Design Problem

A Design Problem (DP) is abstracted as a monotone map h between provided **functionalities** and the *antichain* of requires **resources** (posets $\langle \mathcal{F}, \preceq_{\mathcal{F}} \rangle$ and $\langle \mathcal{R}, \preceq_{\mathcal{R}} \rangle$).

Abstraction of a Co-Design Problem

A Co-Design Problem (CDP) is abstracted as an interconnection of individual DPs.

Co-Design Goal

Find the antichain of all *rational* **resources** $r_1, \dots, r_N \in \mathcal{R}$ which provide a given **functionality** $f \in \mathcal{F}$.

Modeling – Network Flow Model for Intermodal AMoD

Assumptions

- Network flow approach
- Time-invariant model.

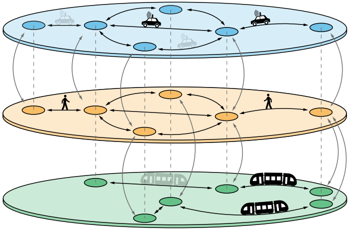
Mesoscopic

System-level planning perspective

$\mathcal{G}_{\mathrm{R}} = (\mathcal{V}_{\mathrm{R}}, \mathcal{A}_{\mathrm{R}})$

$\mathcal{G}_{\mathrm{W}} = (\mathcal{V}_{\mathrm{W}}, \mathcal{A}_{\mathrm{W}})$

$\mathcal{G}_{\mathrm{P}} = (\mathcal{V}_{\mathrm{P}}, \mathcal{A}_{\mathrm{P}})$



Graph

Mode-switching arcs: $\mathcal{A}_{\mathrm{C}} \subseteq \mathcal{V}_{\mathrm{R}} \times \mathcal{V}_{\mathrm{W}} \cup \mathcal{V}_{\mathrm{W}} \times \mathcal{V}_{\mathrm{R}} \cup \mathcal{V}_{\mathrm{P}} \times \mathcal{V}_{\mathrm{W}} \cup \mathcal{V}_{\mathrm{W}} \times \mathcal{V}_{\mathrm{P}}$.
Extended graph: $\mathcal{A} = \mathcal{A}_{\mathrm{W}} \cup \mathcal{A}_{\mathrm{R}} \cup \mathcal{A}_{\mathrm{P}} \cup \mathcal{A}_{\mathrm{C}}$, $\mathcal{V} = \mathcal{V}_{\mathrm{R}} \times \mathcal{V}_{\mathrm{W}} \times \mathcal{V}_{\mathrm{P}}$.

Modeling – Network Flow Model for Intermodal AMoD

Travel Requests

Assume M **travel requests** $\rho_m = (o_m, d_m, \alpha_m) \in \mathcal{V}_\mathrm{W} \times \mathcal{V}_\mathrm{W} \times \mathbb{R}_+, \ \forall m \in \{1, \dots, M\}$.

Constraints

$$\sum_{i:(i,j)\in\mathcal{A}} f_m(i,j) + \mathbb{I}_{j=o_m} \cdot \alpha_m = \sum_{k:(j,k)\in\mathcal{A}} f_m(j,k) + \mathbb{I}_{j=d_m} \cdot \alpha_m, \quad \forall m \in \mathcal{M}, j \in \mathcal{V}$$

$$\sum_{i:(i,j)\in\mathcal{A}_\mathrm{R}} \left(f_0(i,j) + \sum_{m\in\mathcal{M}} f_m(i,j) \right) = \sum_{k:(j,k)\in\mathcal{A}_\mathrm{R}} \left(f_0(j,k) + \sum_{m\in\mathcal{M}} f_m(j,k) \right), \quad \forall j \in \mathcal{V}_\mathrm{R}$$

$$f_m(i,j) \geq 0, \quad \forall m \in \mathcal{M}, (i,j) \in \mathcal{A}$$

$$f_0(i,j) \geq 0, \quad \forall (i,j) \in \mathcal{A}_\mathrm{R}.$$

Modeling – Travel Time and Speed

Road

- Each $(i,j) \in \mathcal{A}_{\mathrm{R}}$ has a speed limit $v_{\mathrm{L},ij}$.
- AVs safety protocols impose a maximum achievable speed v_{a}.
- Too slow AVs are dangerous: (i,j) is kept in $\mathcal{A}_{\mathrm{R}}$ iff $v_{\mathrm{a}} \geq \beta \cdot v_{\mathrm{L},ij}, \quad \beta \in (0,1]$.
- Then, $v_{ij} = \min\{v_{\mathrm{a}}, v_{\mathrm{L},ij}\}$.

Pedestrians

Constant walking speed v_{ij} for each $(i,j) \in \mathcal{A}_{\mathrm{W}}$.

Public Transportation System

- The public transit system at node j operates with frequency φ_j.
- Switching from j to a pedestrian vertex i takes t_{WP}: $t_{ij} = t_{\mathrm{WP}} + \frac{1}{\varphi_j}\ \forall (i,j) \in \mathcal{A}_{\mathrm{P}}$.

Modeling – Properties

Energy Consumption

AVs:

- Urban driving cycle.
- $e_{ij} = e_{\text{cycle}} \cdot \frac{s_{ij}}{s_{\text{cycle}}} \; \forall (i,j) \in \mathcal{A}_{\text{R}}$.

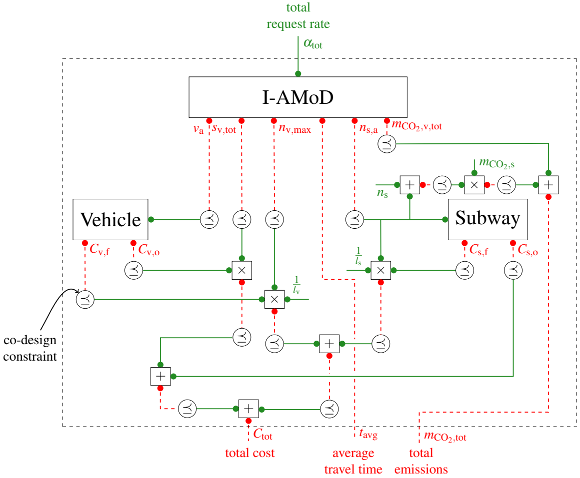
Public Transportation:

- Assumption: Customers-independent operation.
- Constant energy consumption per unit time.

AVs Fleet Size

$$n_{\text{v,e}} = \sum_{(i,j)\in\mathcal{A}_{\text{R}}} \left(f_0(i,j) + \sum_{m\in\mathcal{M}} f_m(i,j) \right) \cdot t_{ij} \leq n_{\text{v,max}}.$$

Co-Design – The Monotone Co-Design Problem



Co-Design – AVs

AVs Design Problem

We design the maximal achievable speed v_a.

Functionality:

- Maximal achievable speed v_a.
- $\mathcal{F}_v = \overline{\mathbb{R}}_+$ (in mph).

Resources:

- Vehicle fixed costs $C_{v,f} = C_{v,v} + C_{v,a}$.
- Vehicle operational costs $C_{v,o}$.
- $\mathcal{R}_v = \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+$ (in USD $\times$ USD/mile)

Functionality/Resources Relation

- Higher speed, more advanced technology.
- v_a as monotone function of costs.

Co-Design – Public Transportation System

Subway Design Problem

We design the service frequency φ_j, assuming

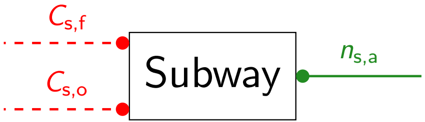
$$\frac{\varphi_j}{\varphi_{j,\text{baseline}}} = \frac{n_\text{s}}{n_{\text{s,baseline}}}.$$

Functionality:

- Acquired trains $n_{\text{s,a}} = n_\text{s} - n_{\text{s,baseline}}$.
- $\mathcal{F}_\text{s} = \overline{\mathbb{N}}$.

Resources:

- Train fleet fixed costs $C_{\text{s,f}}$.
- Train fleet operational costs $C_{\text{s,o}}$.
- $\mathcal{R}_\text{s} = \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+$ (in USD $\times$ USD/year).



Functionality/Resources Relation

- More trains, higher fixed costs.
- More trains require more operators: higher operational costs.

Co-Design – I-AMoD Optimization Framework

I-AMoD Optimization Framework Design Problem

Functionality:

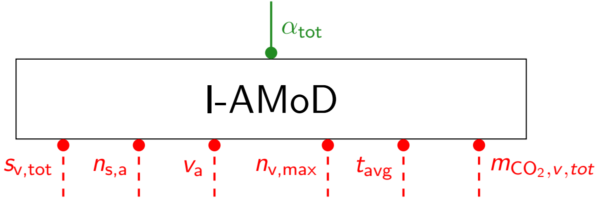
- Demand satisfaction:
  $\alpha_{\text{tot}} := \sum_{m \in \mathcal{M}} \alpha_m.$
- $\mathcal{F}_o = \overline{\mathbb{R}}_+.$

Resources:

- Maximal achievable speed v_a.
- Available AVs per fleet $n_{\text{v,max}}$.
- Acquired trains $n_{\text{s,a}}$.

- Average travel time per trip:
  $$t_{\text{avg}} := \frac{1}{\alpha_{\text{tot}}} \sum_{m \in \mathcal{M}, (i,j) \in \mathcal{A}} t_{ij} \cdot f_m(i,j),$$
- Total AVs-driven distance:
  $$s_{\text{v,tot}} := \sum_{(i,j) \in \mathcal{A}_{\text{R}}} s_{ij} \cdot \left(f_0(i,j) + \sum_{m \in \mathcal{M}} f_m(i,j) \right).$$
- AVs emissions:
  $$m_{\text{CO}_2,\text{v,tot}} := \gamma \sum_{(i,j) \in \mathcal{A}_{\text{R}}} e_{ij} \cdot \left(f_0(i,j) + \sum_{m \in \mathcal{M}} f_m(i,j) \right).$$
- $\mathcal{R}_o = \overline{\mathbb{R}}_+ \times \overline{\mathbb{N}} \times \overline{\mathbb{N}} \times \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+$

Co-Design – I-AMoD Optimization Framework



Functionality/Resources Relation

$$\min_{\{f_m(\cdot,\cdot)\}_m, f_0(\cdot,\cdot)} t_{\text{avg}} = \frac{1}{\alpha_{\text{tot}}} \sum_{m \in \mathcal{M}, (i,j) \in \mathcal{A}} t_{ij} \cdot f_m(i,j), \quad \text{s.t. } (1), (2), (3).$$

- (1): Flows conservation and non-negativity.
- (2): Road congestion.
- (3): Fleet limitations.

Co-Design – The Monotone Co-Design Problem

Full Co-Design Problem

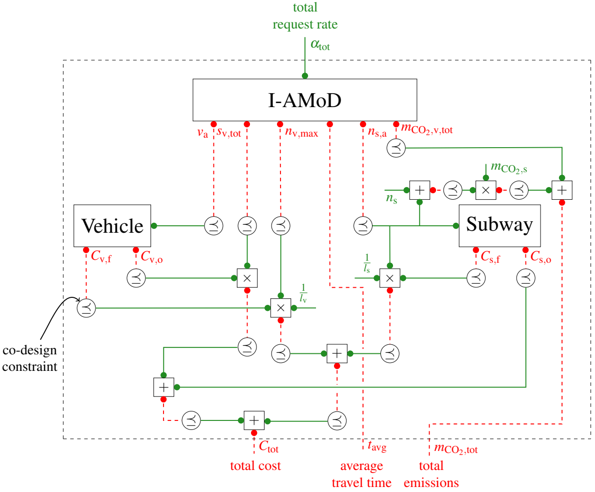
Functionality:

- Demand satisfaction α_{tot}
- $\mathcal{F} = \overline{\mathbb{R}}_+$.

Resources:

- Total costs $C_{\text{tot}} = C_{\text{v}} + C_{\text{s}}$, with
  - $C_{\text{v}} = \frac{C_{\text{v,f}}}{l_{\text{v}}} \cdot n_{\text{v}} + C_{\text{v,o}} \cdot s_{\text{v,tot}}$.
  - $C_{\text{s}} = \frac{C_{\text{s,f}}}{l_{\text{s}}} \cdot n_{\text{s,a}} + C_{\text{s,o}}$.
- Average travel time per trip t_{avg}.
- Total emissions: $m_{CO_2,tot} = m_{CO_2,\text{v,tot}} + m_{CO_2,\text{s}} \cdot n_{\text{s}}$
- $\mathcal{R} = \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+$

Co-Design – The Monotone Co-Design Problem



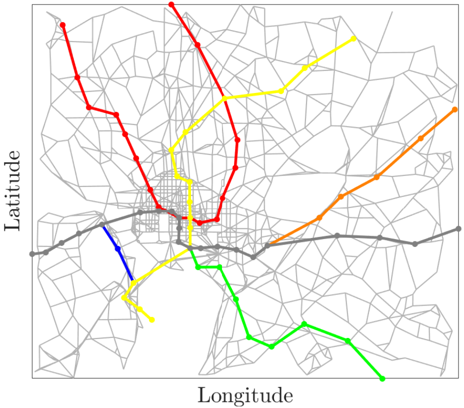
Case Study – Washington DC, USA

Dataset Construction

Road network: OpenStreetMap.
Public Transit network: GTFS.
Origin-destination pairs: WMATA.
Demand: 15,872 travel requests → 24.22 requests/s

Co-Design

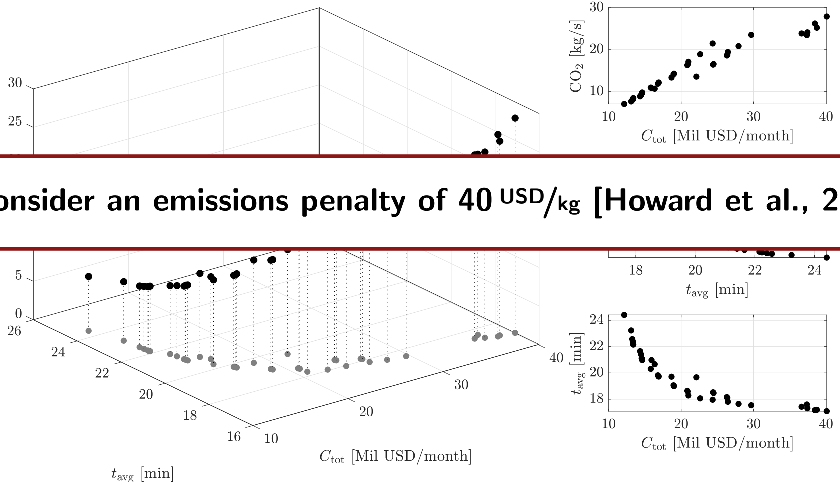
Subway frequency: $\{100\%, 133\%, 200\%\}$.
AVs speed: $v_a \in \{20\,\text{mph}, 25\,\text{mph}, \ldots, 50\,\text{mph}\}$.
Fleet size: $n_{v,\max} \in \{0, 500, \ldots, 6000\}$.



Case Study – Parameters and Units for Sensitivity

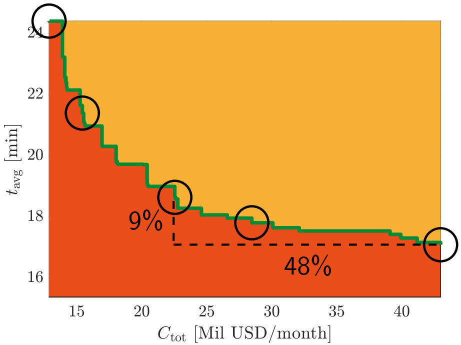
Parameter		**Variable**	**Value**					**Units**
Baseline road usage		u_{ij}	93					%
			Case 1	**Case 2.1**	**Case 2.2**	**Case 3.1**	**Case 3.2**	
Vehicle operational cost		$C_{\mathrm{v,o}}$	0.084	0.084	0.062	0.084	0.084	USD/mile
Vehicle cost		$C_{\mathrm{v,v}}$	32,000	32,000	26,000	32,000	32,000	USD/car
Vehicle automation cost	20 mph	$C_{\mathrm{v,a}}$	15,000	20,000	3,700	0	500,000	USD/car
	25 mph		15,000	30,000	4,400	0	500,000	USD/car
	30 mph		15,000	55,000	6,200	0	500,000	USD/car
	35 mph		15,000	90,000	8,700	0	500,000	USD/car
	40 mph		15,000	115,000	9,800	0	500,000	USD/car
	45 mph		15,000	130,000	12,000	0	500,000	USD/car
	50 mph		15,000	150,000	13,000	0	500,000	USD/car
Vehicle life		l_{v}	5	5	5	5	5	years
CO_2 per Joule		γ	0.14	0.14	0.14	0.14	0.14	g/kJ
Time from $\mathcal{G}_{\mathrm{W}}$ to $\mathcal{G}_{\mathrm{R}}$		t_{WR}	300	300	300	300	300	s
Time from $\mathcal{G}_{\mathrm{R}}$ to $\mathcal{G}_{\mathrm{W}}$		t_{RW}	60	60	60	60	60	s
Speed limit fraction		β	$\frac{1}{1.3}$	$\frac{1}{1.3}$	$\frac{1}{1.3}$	$\frac{1}{1.3}$	$\frac{1}{1.3}$	-
Subway operational cost	100 %	$C_{\mathrm{s,o}}$	148,000,000					USD/year
	133 %		197,000,000					USD/year
	200 %		295,000,000					USD/year
Subway fixed cost		$C_{\mathrm{s,f}}$	14,500,000					USD/train
Train life		l_{s}	30					years
Subway CO_2 emissions per train		$m_{\mathrm{CO_2,s}}$	140					ton/year
Train fleet baseline		$n_{\mathrm{s,baseline}}$	112					trains
Subway service frequency		$\varphi_{j,\mathrm{baseline}}$	$\frac{1}{6}$					1/minutes
Time from $\mathcal{G}_{\mathrm{W}}$ to $\mathcal{G}_{\mathrm{P}}$ and vice-versa		t_{WS}	60					s

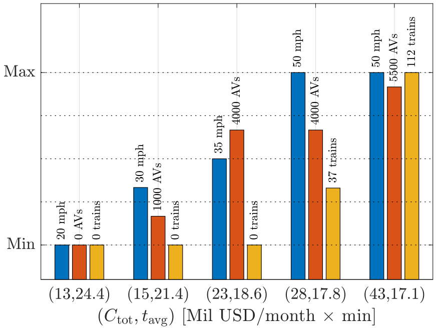
Results – Case 1 (Constant Automation Cost)



We consider an emissions penalty of 40 USD/kg [Howard et al., 2015]

Results – Case 1 (Constant Automation Cost)





Extension – Parking Space Allocation

Autonomous cars could cut traffic and pollution — or make them worse, planners say
(Washington Post, 2019)

Automated vehicles can't save cities
(New York Times, 2018)

Autonomous vehicles: To park or not to park?
(Forbes, 2019)

Extension – Parking Space Allocation

Consider travel requests in two **consecutive** time windows:

$$\rho_{m_1} = (o_{m_1}, d_{m_1}, \alpha_{m_1})\ \forall m_1 \in \{1, \dots, M_1\}, \quad \eta_{m_2} = (o_{m_2}, d_{m_2}, \alpha_{m_2})\ \forall m_2 \in \{1, \dots, M_2\}.$$

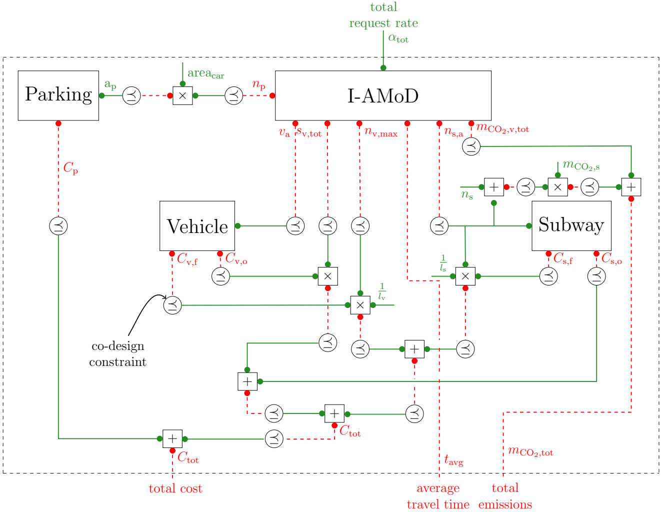
Look at the flow changes on road arcs:

$$\Delta\text{Flow}(i,j) = |f_0(i,j) - g_0(i,j) + f_{m_1}(i,j) - g_{m_2}(i,j)| \quad \forall (i,j) \in \mathcal{A}_\text{R}.$$

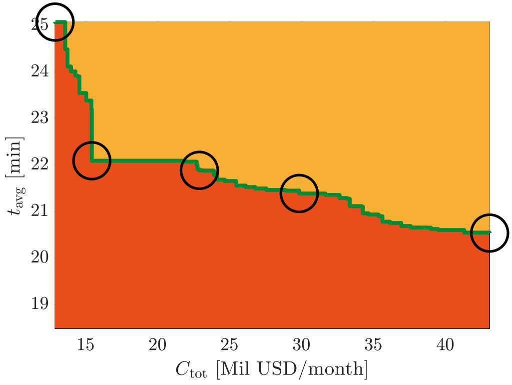
Distribute parked cars accordingly:

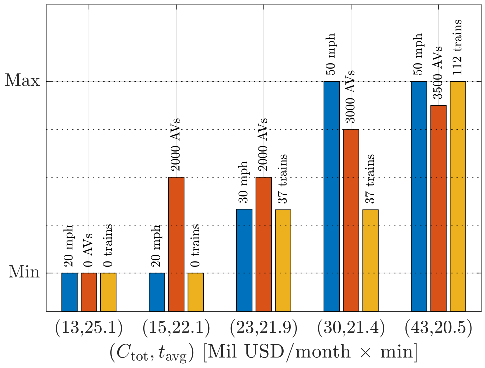
$$n_\text{p}(i,j) = \frac{\Delta\text{Flow}(i,j)}{\sum_{(i,j)\in\mathcal{A}_\text{R}} \Delta\text{Flow}(i,j)} \cdot (n_{\text{v,max}} - n_{\text{v,e}})_\eta$$

Extension – Parking Space Allocation

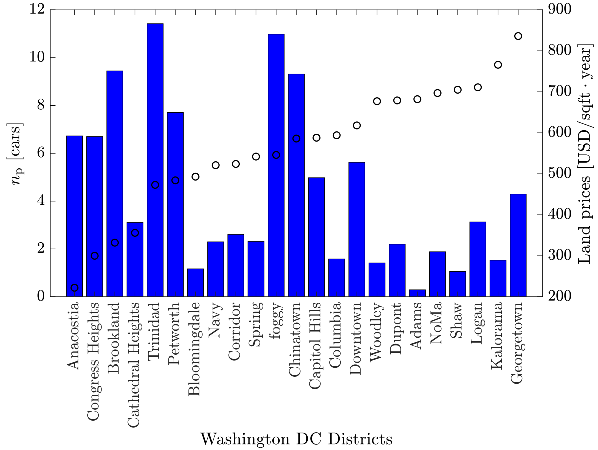


Extension – Case 1 Preliminary Results





Extension – Case 1 Preliminary Results (musical chairs)



Conclusions

Summary

- Co-Design framework for future mobility systems.
- Provides a new, different perspective.
- Tool for stakeholders such as AVs companies and policy makers.
- Modular and compositional, ready to be extended.

Outlook

- Parking space allocation.
- User-friendly interface.
- Model complexity.

Background – Co-Design

Partial Orders

Consider a set $\mathcal{P}$ and a partial order $\preceq_{\mathcal{P}}$, defined as a reflexive, antisymmetric, and transitive relation. Then, $\mathcal{P}$ and $\preceq_{\mathcal{P}}$ define the partially ordered set (poset) $\langle \mathcal{P}, \preceq_{\mathcal{P}} \rangle$.

Bottom and Top

The least and maximum elements of a poset are called bottom and top, and are denoted by $\bot_{\mathcal{P}}$ and $\top_{\mathcal{P}}$, respectively.

CPO and DCPO

A set $S \subseteq \mathcal{P}$ is *directed* if each pair of elements $x, y \in S$ has an upper bound. A poset is a *directed complete partial order* (DCPO) if each of its directed subsets has a top, and it is a *complete partial order* (CPO) if it has a bottom as well.

Background – Co-Design

Chains and Antichains

A *chain* is a subset $S \subseteq \mathcal{P}$ where all elements are comparable, i.e., for $x, y \in S$, $x \preceq_{\mathcal{P}} y$ or $y \preceq_{\mathcal{P}} x$. Conversely, an *antichain* is a subset $S \subseteq \mathcal{P}$ where no elements are comparable, i.e., for $x, y \in S$, $x \preceq_{\mathcal{P}} y$ implies $x = y$.

Monotonicity

A map $g : \mathcal{P} \to \mathcal{Q}$ between two posets is *monotone* iff $x \preceq_{\mathcal{P}} y$ implies $g(x) \preceq_{\mathcal{Q}} g(y)$.

Scott Continuity

A map $f : \mathcal{P} \to \mathcal{Q}$ between directed complete partial orders (DCPOs) is **Scott Continuous** iff for each directed subset $D \subseteq \mathcal{P}$, the image $f(D)$ is directed, and $f(\sup(D)) = \sup f(D)$.

Background – Co-Design

Least Fixed Point

A **least fixed point** of $f : \mathcal{P} \to \mathcal{P}$ is the minimum (if it exists) of the set of fixed points of f:

$$\mathrm{lfp}(f) = \min_{\preceq} \{x \in \mathcal{P} : f(x) = x\}.$$

The least fixed point does not need to exist. Monotonicity of the map f plus completeness is sufficient to ensure existence.

Lemma

- If $\mathcal{P}$ is a CPO and $f : \mathcal{P} \to \mathcal{P}$ is monotone, then $\mathrm{lfp}(f)$ exists.
- Assume $\mathcal{P}$ is a CPO, and $f : \mathcal{P} \to \mathcal{P}$ is Scott continuous. then the least fixed point of f is the supremum of the Kleene ascent chain

$$\bot \preceq f(\bot) \preceq f(f(\bot)) \preceq \ldots \preceq f^{(n)}(\bot) \preceq \ldots.$$

Background – Co-Design

Design Problem

A design problem (DP) is a tuple $\langle \mathcal{F}, \mathcal{R}, h \rangle$ such that $\mathcal{F}$ and $\mathcal{R}$ are CPOs, and $h : \mathcal{F} \to \mathrm{A}\mathcal{R}$ is a **monotone** and **Scott-continuous** function. Each functionality $f \in \mathcal{F}$ corresponds to an antichain of resources $h(f) \in \mathrm{A}\mathcal{R}$.

Monotone Co-Design Problem

A MCDP is a tuple $\langle \mathcal{A}, \mathbf{T}, v \rangle$, where:

- $\mathcal{A}$ is any set of atoms, to be used as labels.
- The term $\mathbf{T}$ in the $\{\text{series}, \text{par}, \text{loop}\}$ algebra describes the scruture of the graph:

$$\mathbf{T} \in \text{Terms}(\{\text{series}, \text{par}, \text{loop}\}, \mathcal{A}).$$

- The valuation: $v : \mathcal{A} \to \text{DP}$ assigns a DP to each atom.

Background – Co-Design

Product Operator

For two maps $h_1 : \mathcal{F}_1 \to \mathsf{A}\mathcal{R}_1$ and $h_2 : \mathcal{F}_2 \to \mathsf{A}\mathcal{R}_2$, define

$$\begin{aligned} h_1 \otimes h_2 : (\mathcal{F}_1 \times \mathcal{F}_2) &\to \mathsf{A}(\mathcal{R}_1 \times \mathcal{R}_2) \\ \langle f_1, f_2 \rangle &\mapsto h_1(f_1) \times h_2(f_2). \end{aligned}$$

Series Operator

For two maps $h_1 : \mathcal{F}_1 \to \mathsf{A}\mathcal{R}_1$ and $h_2 : \mathcal{F}_2 \to \mathsf{A}\mathcal{R}_2$, if $\mathcal{R}_1 = \mathcal{F}_2$, define

$$\begin{aligned} h_1 \bigcirc h_2 : \mathcal{F}_1 &\to \mathsf{A}\mathcal{R}_2 \\ h_1 &\mapsto \operatorname*{Min}_{\preceq \mathcal{R}_2} \bigcup_{r_1 \in h_1(f)} h_2(r_1). \end{aligned}$$

Background – Co-Design

Loop Operator

For a map $h : \mathcal{F}_1 \times \mathcal{F}_2 \to A\mathcal{R}$, define

$$\begin{aligned} h^\dagger : \mathcal{F}_1 &\to A\mathcal{R}, \\ f_1 &\mapsto \mathrm{lfp}\left(\Psi^h_{f_1}\right), \end{aligned}$$

where lfp is the least-fixed point operator, and $\Psi^h_{f_1}$ is

$$\begin{aligned} \Psi^h_{f_1} : A\mathcal{R} &\to A\mathcal{R}, \\ R &\mapsto \mathop{\mathrm{Min}}_{\preceq_{\mathcal{R}}} \bigcup_{r \in R} h(f_1, r) \cap \uparrow r. \end{aligned}$$

Background – Co-Design

DPI

A design problem with implementation (DPI) is a tuple $\langle \mathcal{F}, \mathcal{R}, \mathcal{I}, \text{exec}, \text{eval} \rangle$, where

- $\mathcal{F}$ is a poset, called **functionality** space.
- $\mathcal{R}$ is a poset, called **resources** space.
- $\mathcal{I}$ is a poset, called **implementation** space.
- the map $\text{exec} : \mathcal{I} \to \mathcal{F}$, execution, maps an implementation to the functionality it provides.
- the map $\text{eval} : \mathcal{I} \to \mathcal{R}$, evaluation, maps an implementation to the resource it requires.

Background – Co-Design

Problem

Given a functionality $f \in \mathcal{F}$, find the implementations in $\mathcal{I}$ that realize the functionality f (or higher) with minimal resources, or provide a proof that there are none:

$$\begin{cases} \text{using} & i \in \mathcal{I}, \\ \text{Min}_{\preceq_{\mathcal{R}}} & r, \\ \text{s.t.} & r = \text{eval}(i), \\ & f \preceq_{\mathcal{F}} \text{exec}(i). \end{cases}$$

Background – Co-Design

Problem

Given a DPI $\langle \mathcal{F}, \mathcal{R}, \mathcal{I}, \text{exec}, \text{eval}\rangle$, define the map $h : \mathcal{F} \to A\mathcal{R}$ that associates to each functionality f the objective function of Problem 1, which is the set of minimal resources necessary to realize f:

$$
\begin{aligned}
h: \quad & \mathcal{F} \to A\mathcal{R}, \\
& f \mapsto \operatorname*{Min}_{\preceq_{\mathcal{R}}} \{\text{eval}(i) | (i \in \mathcal{I}) \wedge (f \preceq \text{exec}(i))\}.
\end{aligned}
$$

If a certain functionality f in infeasible, then $h(f)$ is the empty set.

Background – Co-Design Complexity

Suppose $\mathrm{dp}_0 = \mathrm{loop}(\mathrm{dp}_0)$, where dp_0 is an MCDP that is described only using series and parallel operators. Suppose that the resource space is $\mathcal{R}_0$. Then evaluating h_0 takes at most c computation:

- Memory: $O(\mathrm{width}(\mathcal{R}_0))$.
- Number of steps: $O(\mathrm{height}(\mathrm{A}\mathcal{R}_0))$.
- Computation: $O(\mathrm{width}(\mathcal{R}_0) \times \mathrm{height}(\mathrm{A}\mathcal{R}_0) \times c)$

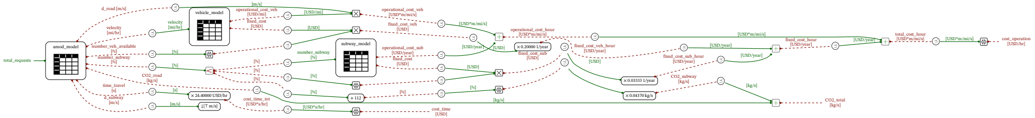
Modeling – Congestion Model

Problem

Each road arc is subject to a baseline usage u_{ij} and has a nominal capacity c_{ij}.

$$f_0(i,j) + \sum_{m \in \mathcal{M}} f_m(i,j) + u_{ij} \leq c_{ij} \quad \forall (i,j) \in \mathcal{A}_\mathrm{R}.$$

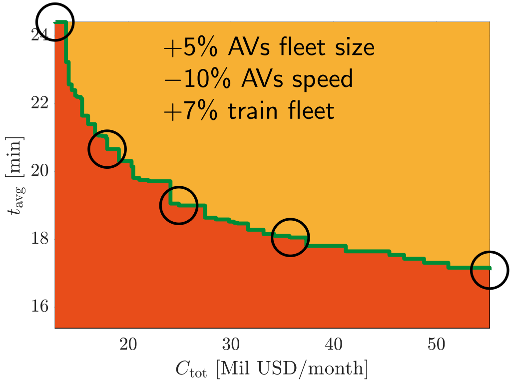
Co-Design – The Monotone Co-Design Problem

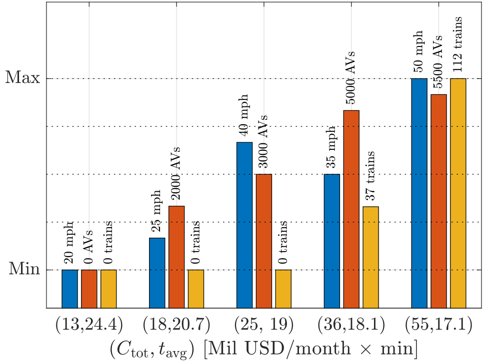


Case Study – Parameters and Units for Sensitivity

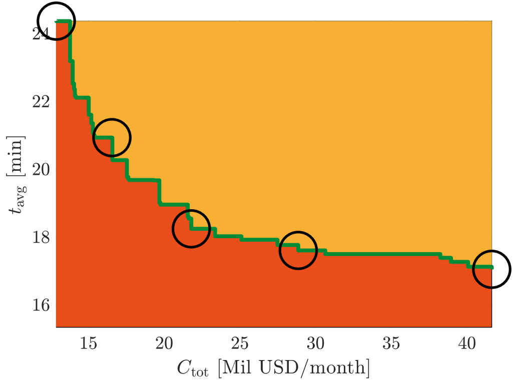
Parameter		Variable	Value					Units
Baseline road usage		u_{ij}	93					%
			Case 1	**Case 2.1**	**Case 2.2**	**Case 3.1**	**Case 3.2**	
Vehicle operational cost		$C_{\mathrm{v,o}}$	0.084	0.084	0.062	0.084	0.084	USD/mile
Vehicle cost		$C_{\mathrm{v,v}}$	32,000	32,000	26,000	32,000	32,000	USD/car
Vehicle automation cost	20 mph	$C_{\mathrm{v,a}}$	15,000	20,000	3,700	0	500,000	USD/car
	25 mph		15,000	30,000	4,400	0	500,000	USD/car
	30 mph		15,000	55,000	6,200	0	500,000	USD/car
	35 mph		15,000	90,000	8,700	0	500,000	USD/car
	40 mph		15,000	115,000	9,800	0	500,000	USD/car
	45 mph		15,000	130,000	12,000	0	500,000	USD/car
	50 mph		15,000	150,000	13,000	0	500,000	USD/car
Vehicle life		l_{v}	5	5	5	5	5	years
CO_2 per Joule		γ	0.14	0.14	0.14	0.14	0.14	g/kJ
Time from $\mathcal{G}_{\mathrm{W}}$ to $\mathcal{G}_{\mathrm{R}}$		t_{WR}	300	300	300	300	300	s
Time from $\mathcal{G}_{\mathrm{R}}$ to $\mathcal{G}_{\mathrm{W}}$		t_{RW}	60	60	60	60	60	s
Speed limit fraction		β	$\frac{1}{1.3}$	$\frac{1}{1.3}$	$\frac{1}{1.3}$	$\frac{1}{1.3}$	$\frac{1}{1.3}$	-
Subway operational cost	100 %	$C_{\mathrm{s,o}}$	148,000,000					USD/year
	133 %		197,000,000					USD/year
	200 %		295,000,000					USD/year
Subway fixed cost		$C_{\mathrm{s,f}}$	14,500,000					USD/train
Train life		l_{s}	30					years
Subway CO_2 emissions per train		$m_{\mathrm{CO_2,s}}$	140					ton/year
Train fleet baseline		$n_{\mathrm{s,baseline}}$	112					trains
Subway service frequency		$\varphi_{j,\mathrm{baseline}}$	$\frac{1}{6}$					1/minutes
Time from $\mathcal{G}_{\mathrm{W}}$ to $\mathcal{G}_{\mathrm{P}}$ and vice-versa		t_{WS}	60					s

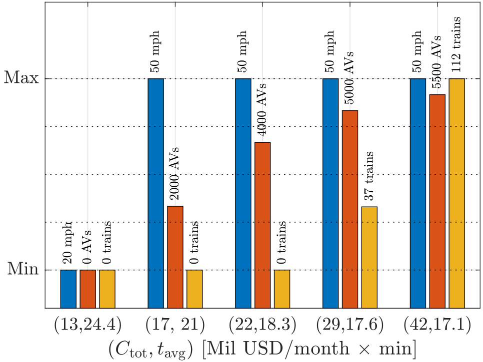
Results – Case 2.1 (speed-dependent automation cost)



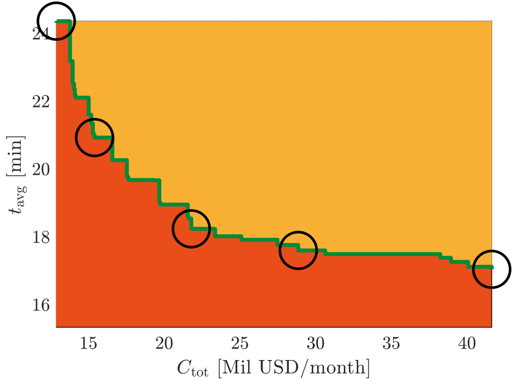


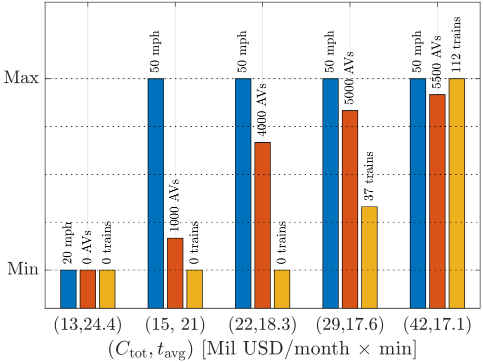
Results – Case 2.2 (speed-dependent automation cost)



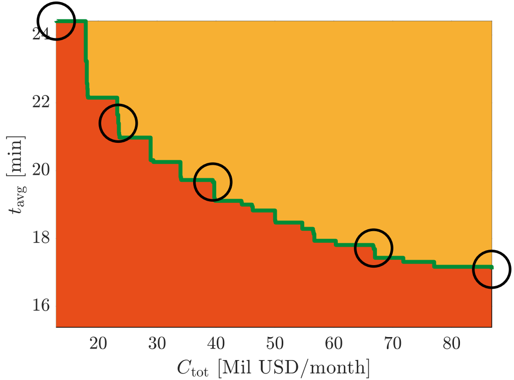


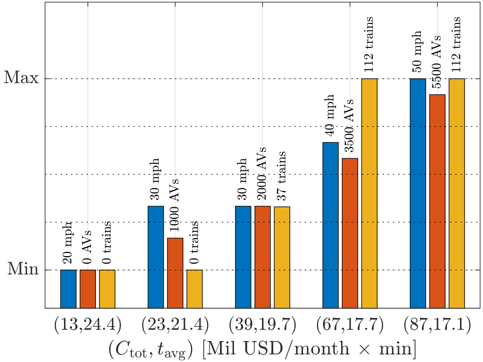
Results – Case 3.1 (no automation cost)





Results – Case 3.2 (high automation cost)





Results – Sensitivity Analysis